# Some Physical Properties of the Modern Violin Bow

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**Abstract.** The modern violin bow—or more generally: any modern bow belonging to an instrument of the violin family—possesses remarkable properties in terms of potential for manipulation of timbre, dynamics, articulation, and envelope. While *timbre* and *dynamics* are strongly related to the player's choice of speed, force, and position on the string, *articulation* and *envelope* are often built around of the bow's intrinsic dynamic properties, e.g., the mass and springiness of bow hair and stick, allowing the player to mainly *initiate* a development that subsequently will be unfolding without too much interaction.

## 1 On the Development of the Modern Violin Bow

The ancient string instruments were either *plucked*—with or without a plectrum; *struck*—with some kind of stick or mallet; or *bowed*—either with a rosined stick of softwood, or with a rosined string of hair (sometimes other materials) tied to an wooden arc (Ullreich 1998). In the latter case, the bent stick would be strong and resilient enough for tensioning the hair string attached to each stick end, or, more conveniently, asymmetrically so that one overshooting end was left as a "handle" (see Fig 1 a—where the optional handle is shown dotted, and b—which is a further development of the latter concept).

The arc would normally be quite high to ensure a sufficient hair tension in spite of the hair length varying with humidity, etc. Anyone trying to play with a high-arced bow will experience how exhausting it is to keep the arc from rolling towards the strings. A handle might help, however, and by reducing the arc, i.e., lowering its top point with respect to the hair, the balance problem could have been be reduced further, if not for the problem with keeping the hair tension stable. The hair would be fastened to the bow stick with different types of knots, and was sometimes thread through holes in the stick.

Around 1500 the straight bow stick was introduced by Giovanni Bellini (see Fig. 1 c), facilitated by the "frog" introduced a couple centuries earlier—although yet without an adjustment for resolving the hair-tension problem. The hair of bows with straight or low-arced sticks would normally be tensioned by one or more fingers during playing.

The introduction of a bow "head" (see Fig. 1 d, on the right side), increased the distance between stick and hair bundle at the tip end; at the same time a head would be big enough for hiding the knot. With this bow profile, the possibility of simultaneously playing more than two strings *dynamically* became an option, just because hair tension could be varied by use of the bow-hand fingers: With profiles like in Fig. 1 a and b, chords composed of three tones or more can only be played quite loudly, and— for reasons to be explained later—even more so with modern bows, where the tension of the hair ribbon is fixed by a screw controlling the frog position. For more information on the history of the bow, read e.g., W. Bachmann (1964) and Ullreich (1998).

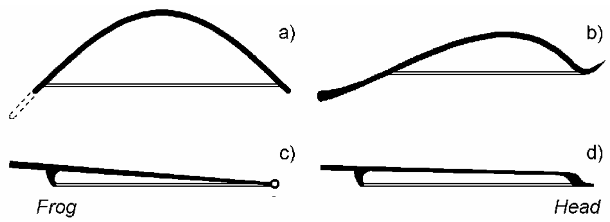


**Figure 1:** Some important steps in the development of the modern bow: (a) The first musical bows were probably Arabian, documented in pictures and manuscripts from the 10th (or possibly the 9th) century. (b) As result of Islamic influence, variations on the Arabian bow were found throughout Europe from the 11th or 12th century. (c) The Giovanni Bellini bow from around 1500 had a straight stick and used a “frog” to keep the hair a distance from it. The *frog* had been introduced ca 1300 on bow models like (b). One novelty of the Bellini bow was the spherical cap covering the knot at the tip. (d) Soon after 1500 bows were equipped with a “head” covering the knot, and with a straight or moderately convex stick. The bows could be held overhandedly or underhandedly, but most often underhandedly for low-pitched instruments.

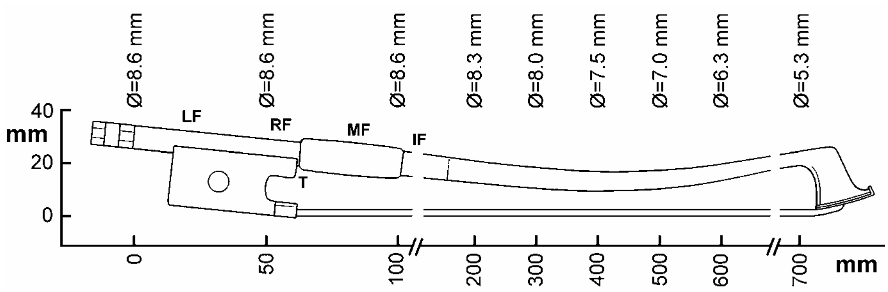


**Figure 2:** A modern violin bow. The design of the modern violin bow is for a great part ascribed to Françios Tourte (Paris, ca 1747–1835). The arc is now concave. The first 10-11 cm of the stick has a uniform diameter, while further out a carefully chosen gradient controls the diminishing thickness all the way to the head. The hair is pressed into a ribbon by means of a D-shaped ferrule and a plug at the frog’s lower-right corner. At the bow’s head, the hair ribbon is led through a square opening in the *tip*—a thin plate of metal, ivory or other material that glued onto the head will prevent the thin wooden walls from breaking under the relative high hair tension used (typically 50 to 65 N before playing). Inside the stick’s rear end, a screw—its cap termed the button—holds the frog, which is gliding in a narrow slot, deeply cut into the wood. Around the stick in front of the frog is usually wrapped a leather pad—the grip—followed by metal windings (e.g., gold or silver), partly to balance the bow, partly to prevent the stick from wearing out. LF, RF, T, MF, and IF indicate approximate position of fingers (see text).

## 2 Some Geometrical Aspects

With François Tourte came the concave profile (camber) of the bow stick. Notice that the stick is not carved to this profile, but is heated and bent after carving (allowing the camber to be re-adjusted at a later stage if desirable). The reduction in transverse bow-hair compliance from a bow with straight stick is substantial—not to speak about comparison to earlier models. With the concave stick, shortening of the distance between frog and tip is mainly dependent on the head's resistance to backward rotation when pressure is applied to the hair ribbon. This means of course that the diameter and gradient of the stick near the head is of uttermost importance. The stick diameters shown in Fig. 2 are derived from a graph by R. Hopfner (1998) and show a gradient, supposedly utilized by François Tourte and later reported by Jean-Baptiste Vuillaume. A closer look at this curve shows that after a cylindrical section of length ca 105 mm, the diameters fall into a nearly straight line if using the *square of the distance* from frog end for abscissa.

Octagon cross sections are often seen in this part of bow stick, up to the head. In fact, most bows have been octagon during early stages of carving in order to better control the gradient. Whether or not the final bow will take an octagon or circular shape (oval have also been utilized) partly depends on how much work the bow maker is prepared to put into the stick, as an octagon finish requires more work for achieving a perfect result with correctly reflecting surfaces of equal widths along the stick. As far as stiffness versus mass is concerned, the octagon (at any angle) surpasses the circular bow by a mere (theoretical) 0.2 %, and should not be of concern for playing quality.

The head is in many ways the signature of the bow designer. It could be mainly rectangular, parallelogram, or triangular and drawn out in a point, etc. Its mass, being critical for the balance and moment of inertia, is indeed a part of the design, but can, if desired later, be adjusted up somewhat by introducing a tiny piece of led in the head's cavity, next to the hair knot. E.g., for double bass bows small pellets are sometimes inserted in order to adjust these parameters and move the point of spiccato towards the tip.

The design of the frog can vary substantially with respect to mass, shape, materials, and decoration. As we shall see in section 4, its mass and weight distribution play a substantial role in both balance and moment of inertia. The same applies to the frog screw with cap, which can vary substantially in length and choice of material.

Tourte replaced the hair *bundle* with a hair *ribbon*. The advantage of shaping the hair into a ribbon is that one gets a more uniform hair tension, implying a more direct string impact with less damping and greater output brilliance; the latter due to better synchronization of slips across the hair width. For the modern violin bow the distance between the ribbon and the stick is typically to 17-19 mm, which is just enough for keeping the cambered stick clear off the string when the bow is bouncing or played vigorously.

Concerning the bow grip, Fig. 2 shows approximate finger positions: The thumb (T) is placed right on the upper-right corner of the frog, or sometimes slightly more towards the leather pad. (Some players prefer to place the thumb further back, inside the frog's curve—this is particularly true for cello and double-bass players). In modern off-string techniques like ricochet and spiccato, the thumb acts largely as an axis with the bow's tip at the circular periphery. The index finger (IF) is usually placed over the windings just before the leather pad, the middle finger (MF) over the pad (sometimes called the *grip*), and the ring finger (RF) over the stick just behind the pad. The index finger is, together with the thumb, the main finger to give the bow an appropriate torque. The ring finger controls the bow-hair tilt by touching the frog just behind the circle (called the *eye* or *pastille*) drawn in the figure. On the violin and viola bows the tip of the little finger (LF) touches the stick just above the frog's heel, and is used for lifting the bow or reducing the bow force when playing soft or close to the frog. On cello and double bass, the little finger is normally positioned a little further down (i.e., touching, or near, the frog). The middle finger has a more limited function.

When used for early music, many players prefer holding the modern bow a few centimeters up the stick (towards its middle), as this reduces the bow's moment of inertia—see Eq. (2) sect. 4—and thus counteracts crisp and direct impacts on the string, which was the achievement of the modern-bow design.

## 3 Choice of materials

The wood almost exclusively chosen for professional-quality bow sticks is Pernambuco (*Guilandia echinata* of family *Caesalpinia*, or synonymously: *Caesalpinia echinata*), a dark red, high density, high Young's-modulus type of wood growing in the northern and middle parts of east Brazil. (Brazil, by the way, literarily means "red wood".) Tourte experimented with a lot of materials for bows, including steel, but concluded with Pernambuco as being the superior choice due to its rare combination of density and stiffness. (With any given homogeneous material: for a doubling of a stick's diameter, its mass and stiffness will increase by factors 4 and 16, respectively. Furthermore: for any given combination of mass and bending stiffness, there is a *diameter* given with each density, and fixed ratio between the density and the product of Young's modulus and this diameter squared .) Of other materials in use, snakewood (*Brosimum guianense*, or synonymously: *Piratinera guianensis*—of red-violet to light-brown color) is the most prominent one, although mainly used for production of historical bows. Compared to Pernambuco, it has typically about the same Young's modulus, but a density some 20 and 40% greater (calling for slimmer bows). Snakewood is easily recognized by its characteristic wavy opalescence, with typical "wavelengths" of 8 to 16 mm.

Of more modern materials, carbon fiber is the most promising one, but a frequent comment (to be believed or not) is that such bows lack nuances in dynamical playing. Nevertheless, with carbon fiber you have control over the mass/stiffness ratio, since you can make such bows hollow. In some of these bows, you can even adjust the bending stiffness manually by means of an internal rope, made of Kevlar, its tension easily adjustable through the frog's screw cap (*button*).

The *tip* is not a term for the pointed end of the stick, but describes the thin plate glued onto the stick's head in order to reinforce its thin walls surrounding the square cavity, inside of which the hair knot is plugged and hidden. The tip was traditionally cut from ivory, but since nowadays import restrictions apply, mammoth has become the more commonly used material. However, for bows with gold wrapping, golden tips, buttons and frog lids are often seen. Silver is also used, and for the tip: synthetic, the latter mostly for students' bows, though. Between the tip and the head is a layer of ebony or ebonite.

The frog is usually made of fine-grain ebony with parts of metal and/or mother-of-pearl (*nacre*). Its design can vary a lot both with respect to shape, materials (François Tourte carved frogs in turtle shell). The button is commonly made of silver combined with ebony and nacre, or from silver or gold alone. German silver (an alloy composed of copper, zinc, and nickel, with possible elements of lead and tin) has also frequently been used for all visible metal parts of the frog. The internal screw is usually made of steel, but a strong light alternative exists in titanium. Through the ferrule (*ring* or *band*) the hair ribbon enters the frog squeezed in by a wedge of cedar, and ends in the frog cavity (*groove*), where its knot is held firm by means of a birch wedge. (A third kind of wood is commonly used for head's wedge, namely linden.) Covering the groove is a lid to slide in from the ferrule side, most typically made of nacre.

The hair is horse tail, preferably from stallions, because urine seems to make the hair more brittle, as well as discoloring it. The bulk of bow hair comes from Mongolia, Siberia, Canada, and China. The main reason for using hair rather than some other material (for example synthetic) is that hair possesses chemical qualities that bind the rosin well, ensuring an even layer after rosin has been applied and distributed through playing. It is often claimed that good bow hair possesses "scales that grip the string well"; some players even prefer their hair scales pointing towards the tip (thus "making up bows easier"), but as far as friction goes: scales has very little to do with it, since they rise up above the hair stem less than 0.5 μm (Rocaboy 1990), which is negligible compared to the string diameter, and moreover they are completely covered when rosin has been applied and partly melted (see Fig.3). But, more importantly: it is the *difference* between sliding friction (with softened rosin) and maximum static friction (with set rosin) that render possible the necessary wave-shaping slip-stick contact with the string. Anyone who has tried playing with an un-rosined bow hair has experienced the impossibility of producing a normal tone. In the literature, maximum temperature differences of rosined hair, caused by slip-stick dynamics, have been reported from 5–6º C (Askenfelt 1995) to 10–25º C (Pickering 1991), all measurement done with infrared camera.

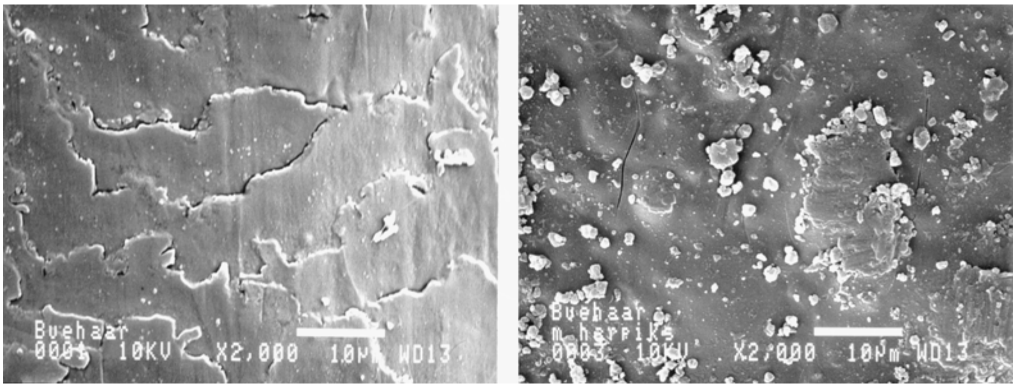


**Figure 3:** Bow hair without rosin (left panel) and with rosin (right panel). Original magnification: 2000 times, the white line indicating 1.0 μm. Scales can be clearly seen in the left panel, but are completely covered by rosin in the right panel. Every time the string slips on the bow hair, the rosin softens in the near vicinity; at the same time small particles of hard rosin are thrown around (seen as small lumps in the right figure). While the string is sliding, stochastic noise is created. (Photos by Norwegian Inst. of Technology, Trondheim)

Before mounting, the hair bundle is wrapped with cotton thread at each end, and a short length of excess hair powdered with rosin and melted into a tiny lump to prevent individual strands to slide out. After mounting, loose hairs can be tightened by gentle exposure to heat.

## 4 The Bow in Numbers and Equations

In this section I will go through a number of equations and parameters relevant for bow-property calculations. As a remainder, I will first present the concept *moment of inertia*, which is a measure for how much a body opposes a rotational acceleration around a given axis. The formula reads:

$$J = \sum_i M_i x_i^2,$$

where

$M_i$ is the infinitesimally small mass at a point $i$, with distance $x_i$ from the axis of rotation;

$J$ is the *moment of inertia* with dimension $\left[\frac{\text{kg m}^2}{\text{rad}}\right]$.

As we shall see, together with the bow's mass, this quantity plays a role of uttermost importance in off-string techniques (such as spiccato and ricochet), where an effective axis exists near the thumb on the frog, as well as in general bow dynamics. The easiest way of finding the moment of inertia with a given axis (e.g., inside the curve of the frog) is to suspend the bow from a thin, round nail in axis position, and measure the bow's pendulum frequency. *J* can thus be calculated as

$$J = m\overline{r}\frac{g}{\omega^2} \approx m\overline{r}\frac{0.2484\,\text{m/s}^2}{f^2},$$

where

$m$ is the total mass of the object; $\overline{r}$ is the distance from axis to the center of gravity;

$g$ is the acceleration of gravity ($\approx$ 9.807 $\text{m/s}^2$); $\omega$ is the angular pendulum frequency,

$f$ is the pendulum frequency in Hz.

Normal values for frequency and moment of inertia when measured that way would be (e.g.) $f$ = 0.73 Hz, giving $J$ = 0.0053 kg $\text{m}^2$/rad, when measuring a 60 gram bow with a distance from its axis to its balance point of $\overline{r}$ = 190 mm.

This routine for measuring the moment of inertia also gives us another important parameter: the *point of percussion* (*PoP*). *PoP* can be compared to the “soft spot” or “sweet spot” of a tennis racket or baseball bat. When hitting the ball in that spot, all rotational energy from the racket/bat is transferred to the ball. This means there will be no (rotational) rebound in the hand of the player, who would otherwise have felt the ball striking back. The distance between the axis and the point of percussion, $l_{PoP}$, can be conveniently calculated as

$$l_{PoP} = \frac{g}{\omega^2} \approx \frac{0.2484\,\text{m/s}^2}{f^2},$$

which in the present case leads to a point 466 mm from the axis, or a point on the stick some 210 mm from the tip when applied to a violin bow of normal length (ca 740-750 mm from cap to the tip point). Notice: this corresponds to the motion of a simple pendulum (point mass) with length $l_{PoP}$; furthermore: for a compound pendulum consisting of a straight rod of uniform cross section and suspended from one end, $l_{PoP}$ will be 2/3 of its length, a fraction which is matched relatively well by the violin bow with respect to the frog axis. On a double-bass bow, however, *PoP* will be found relatively closer to the head. After a series of ricochet tones (see sect 4.4, and the chapter "The Violin Bow in Action—"A Sound-Sculpturing Wand"), *PoP* is the spot where you *do not want to end*, as there will be no rebounding energy to dampen from the impact on the string. The bow is likely to continue bouncing, not in a rotational mode, which could have been dampened directly, but in a translational mode with the whole stick jumping up in parallel. In a rotational mode vibrations can be damped simply by complying with the rebounding action at the frog, which is *up* or *down* dependent on whether the ricochet series ends *inside* or *outside* of *PoP*, respectively. Before going further on this we should look at the bow's separate parts in some detail.

### 4.1 The Stick

Pernambuco—a trade name for the heart wood of the tree—comes with densities between 790 and 1090 kg/m$^3$ (Holtz 1998), the most suited species showing densities around 970-1000 kg/m$^3$ with preferred Yong's modules between 20 and 22 GPa. It is not clear which role the damping plays for quality (typical Q-values range between 80 and 250 in the finished stick); in any case the overall relative damping almost doubles after hair is mounted, and rises with a factor around 20 for a handheld bow. With the stick freely suspended the transverse modes typically take values around 60, 160, 300, 500, 750, 1000, 1300, and 1700 Hz in the lower range (quite comparable to the free-free bar). With frog and hair mounted a modest shift upwards (< 7%) is observed, and a new resonance appears around 60 - 75 Hz (Askenfelt 1995). To the degree these resonances are excited during playing, the bow will act as an absorbent with respect to the transversal and torsional modes of the string (the bow coupled in mechanical series). The weight of the stick without frog and hair is typically 36 to 41 grams, of which the silver/gold winding may count for some 4 - 5 g.

One common way of estimating the stick's bending stiffness is to support it near the frog and head, and measure the load required for a given deflection at a given point (e.g., its middle). Askenfelt (1995) reports the force required for "a student's bow of Chinese origin" to deflect 1.0, 2.0, and 3.0 mm, to be 0.77, 1.49, and 2.26 N, respectively, when loaded near its middle. With the hair tightened to normal tension (ca 60 N), the respective loads were 0.77, 1.26, and 1.85 N, i.e., noticeably *less* that with the bare stick, which is logical when considering that the stick looses some of its camber (becomes straighter) when tightening the hair. The camber, or maximum deflection compared to a straight stick, would normally be some 14-16 mm before hair tensioning, and about half of that afterwards.

### 4.2 The Frog and the Wrapping

Apart from the functional parts of the frog, i.e., the screw, the ferrule, and the enclosed chamber for hair knot and plug, the frog has been open for quite a bit of decorative designs trough the years after Tourte, e.g., with the space between the stick and the hair ribbon open, only separated by twining metal ornaments. However, the modern one-block frog weighs typically 15 – 19 grams, the screw included. Nevertheless, one should remember that together with the stick's leather pad and silver/gold windings, theses parts are very convenient for small alterations of a bow's balance in a nondestructive way. The choice of diameter for the metal winding; how many turns; how much hidden under the leather, etc, all giving room for fine adjustments of the balance point, and equally important—the preferred point of spiccato for any given tempo: By adding weight near the frog, the spiccato point and possibly the effective axis are away from the tip.

### 4.3 The Hair

How many hairs to include in the hair ribbon is a matter of considerable concern, since this too influences quite a few parameters of the bow's playing properties. Because all hairs in contact with the string should be tensioned to ensure consistent friction, less might be better than more. Since a bow looses some of its camber, and thus some of its active stiffness, when tightening the hair, one should consider the influence on camber when deciding the amount. The normal amount of hair for a violin bow ranges between 140 and 200 strands, giving an approximate weight between 4.2 and 6.0 g (with 0.03 g per hair). The stiffness of hair has been estimated to about 0.2 N/mm for a single hair of normal length (650 mm). A normal bow-hair tension of 60 N thus stretches the hair ribbon some 1.5 to 2.1 mm. Individual hairs start deforming when the applied tension surpasses 4 N, and break above 4.5 N, their compliance being relatively linear up to 2.5 – 3.0 N. According to Askenfelt (1995), the compliance is comparable to that of a high-quality fly-fishing leader of diameter 0.20 mm. The normal hair diameter is 0.20 ± 0.5 mm, which is some 2-5 times higher than a human hair.

Longitudinal propagation speed of the hair lies around 2300 m/s, while transverse ditto lies around 90 m/s with hair tension 60 N. An accelerometer mounted at the frog or tip would register slides (hairs vibrating) with fundamental frequencies between 70 and 500 Hz for strokes performed over the entire bow length. Luckily, such disturbing resonances have little chance of getting transmitted through strings and instrument body for finally reaching the listener.

When tensioning the hair, the bow's head is pulled some 1 mm backwards, partly by reduction of the camber, partly by local bending of the stick's most narrow part. This head action is quite noticeable in actual playing: the transverse deflection of bow hairs pressing down a string with a given force is somewhat four times higher near the tip than near the frog, but of course, the transverse compliance is at highest about midway between.

### 4.4 Off-string Techniques and the Bouncing rate

In off-string techniques the bow is rotating around an axis close to the player's thumb (approx. where the hair starts). Knowing the bow's moment of inertia it is straightforward to calculate the bouncing rate for different positions along the hair length (the hair presumed to remain in contact with the string):

$$f_{BNC} = \frac{1}{2\pi}\sqrt{\frac{T_H \dfrac{L_H\, x_H}{L_H - x_H}}{J}}, \qquad \left(\begin{array}{l}\text{where } T_H \text{ is hair tension;}\\ \quad L_H \text{ is total hair length;}\\ \quad x_H \text{ is string's distance from axis.}\end{array}\right)$$

This formula works well for 0.1 m $\leq x_H \leq$ 0.5 m (giving $f_{BNC}$ values from 6 to 25), but will be giving too high rates above 0.5 m if $T_H$ is not reduced to compensate for the head action referred to in section 4.3. However, the string is also compliant and should be included in the equation:

$$f'_{BNC} = \frac{1}{2\pi}\sqrt{\frac{T_H T_S L_H x_H^2}{J\left[L_H L_S T_H (1-\beta)\beta + x_H T_H (L_H - x_H)\right]}} \qquad \left(\begin{array}{l}\text{where subscript } S \text{ indicates "String"}\\ \text{and } \beta \text{ is the relative bowing point}\\ \text{with respect to the bridge distance and } L_S.\end{array}\right)$$

String tension ($T_S$) varies typically from 80 N at the E-string to 45 N at the G-string. When calculating for a typical spiccato point ($x_H$ = 35 cm) and a string $\beta$ of 0.12—i.e., some 4 cm away from the bridge if playing the open string—we get bouncing rates of 13.7 and 13.0 Hz for the E-string and G-string, respectively, which should be compared to the $f_{BNC}$ = 14.7 Hz for the inflexible string of (Eq. ?). E.g., for a fast spiccato one might have to move the bow out some 17 mm to get the same action ($f_{BNC}$) when crossing from the E-string to the more than 6 times heavier G-string. Dependent on the stick profile, this could promote excitation of less desirable bow modes (see the figure below):

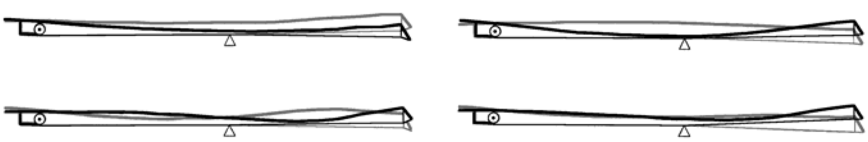


**Figure 4:** Typical bow modes during spiccato (from Askenfelt & Guettler 1998). Pilot studies performed at Royal Swedish Institute of Technology suggest that during a good crisp spiccato the bow is predominantly oscillating in the first mode (head and stick midpoint moving in phase). Other modes might interfere with the regularity of bow-string contact. In these figures the hair ribbon is not part of the modal analysis.

In spiccato and ricochet the bow leaves the string in a short interval, during which gravity and a downward push by the index finger force the bow back onto the string. The resulting spiccato frequency should be close to but not greater than $f_{BNC}$ to avoid excitation of higher bow modes. A good spiccato bow provides sufficient bow force during string contact without requiring much finger action from the player at each impact; it offers clean first-mode oscillations with low damping.

However, if for finding the adequate bounce rate the contact point has to be moved out 5% with respect to the axis, a 5% greater angular velocity at impact is required to maintain the maximum contact force. Low-pitched violin strings have low tension, but high wave resistance—the latter requiring *greater* contact force during attacks—so here is an obvious tradeoff between optimal $f_{BNC}$ and force. But again: some bows handle this better than others… More research is required to fully understand how.

The dynamic properties of the bow are not only used for off-string techniques, they are indeed utilized for articulation of attacks in general. For example: by giving the bow an impulsive torque when starting the stroke, one gets at least a half (damped) cycle of force modulation with the frequency $f_{BNC}$, which we remember is quite low close to the frog. Or, when starting the stroke with the maximum force required and then quickly releasing the torque, a smooth transitional period duration $>0.25/f_{BNC}$ comes for free. That way accents can be made rather effortless, and with different durations at different parts of the bow. It is the benefit of the modern violin bow that this can all be done with minimal movements.

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